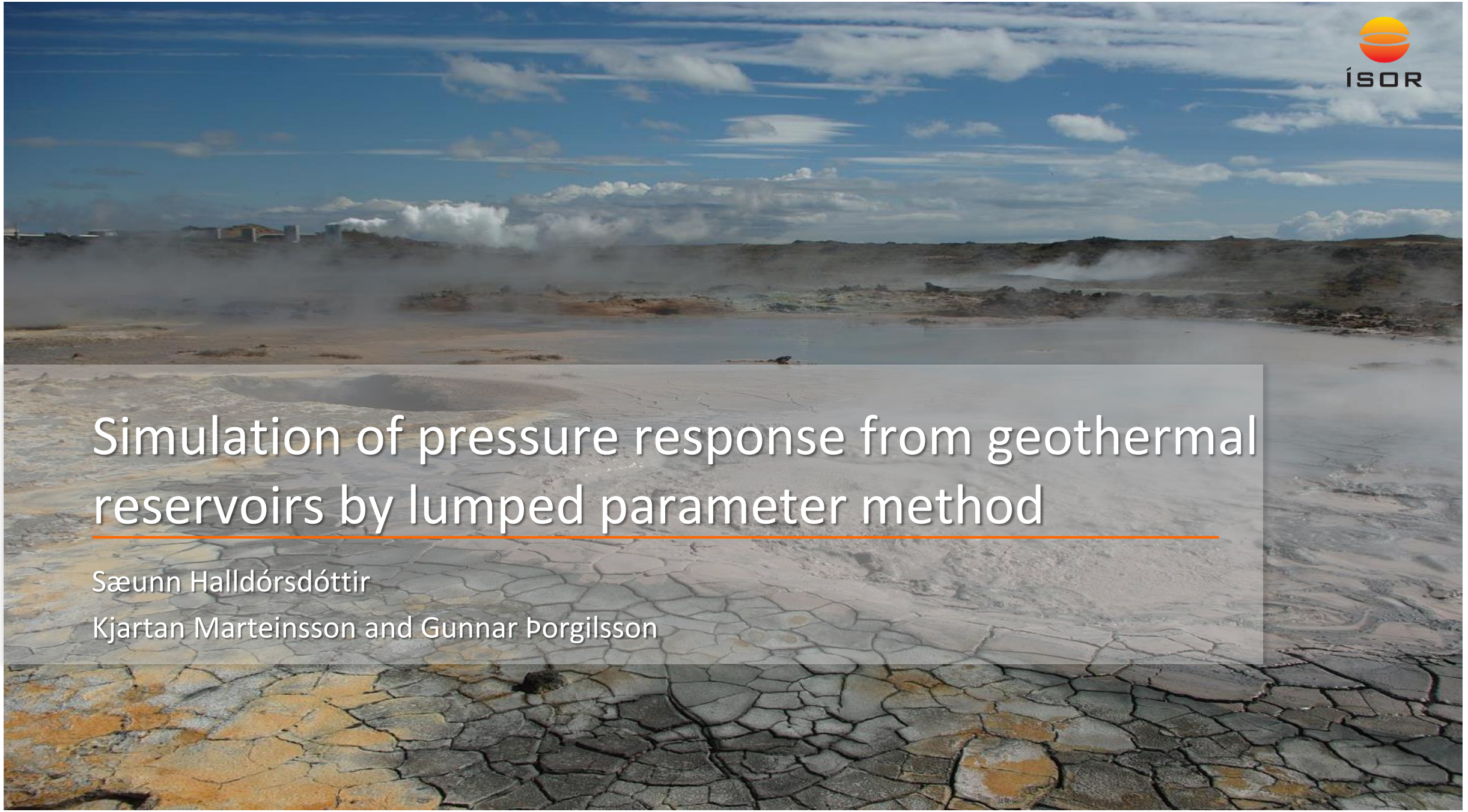


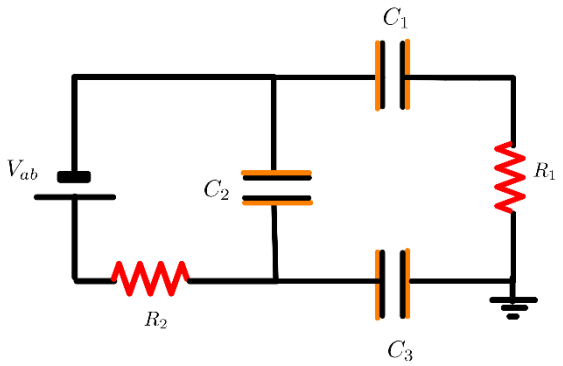
Simulation of pressure response from geothermal reservoirs by lumped parameter method

Sæunn Halldórsdóttir

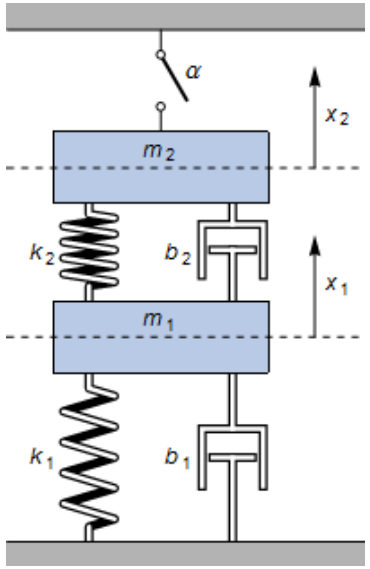
Kjartan Marteinsson and Gunnar Þorgilsson

Lumped parameter models

- A lumped parameter model reduces the description—configuration, geometry, and properties—of a physical system to a topology of a distributed companion system, where properties are integrated into lumped values.
- Response given by analytical functions.



Electric circuits



Mechanical system

Methods in applied geophysics

TABLE I

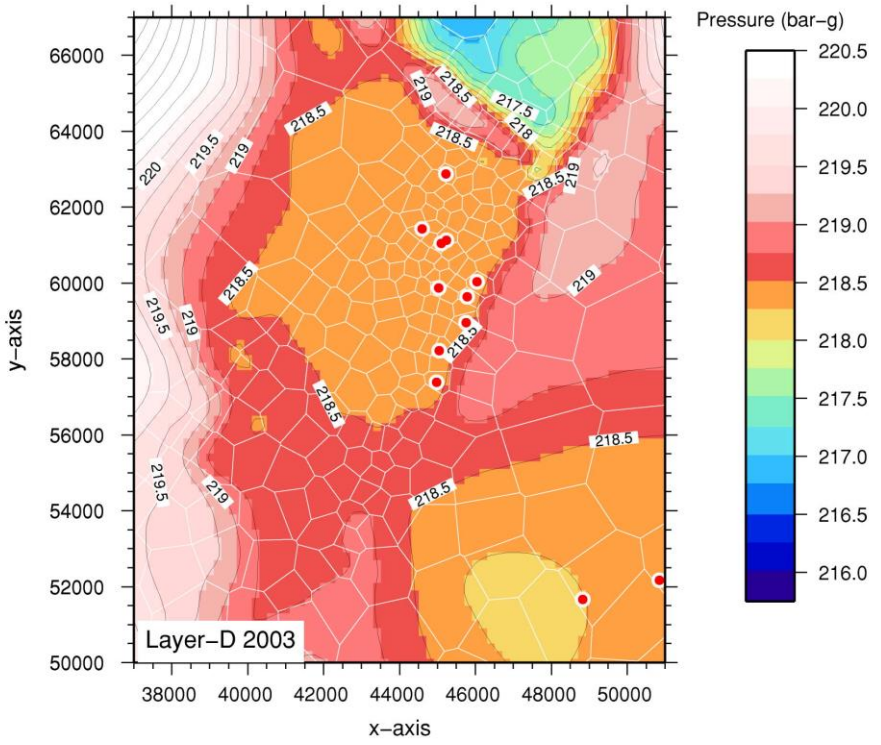
ANALOGIES FOR PARABOLIC DIFFERENTIAL EQUATIONS

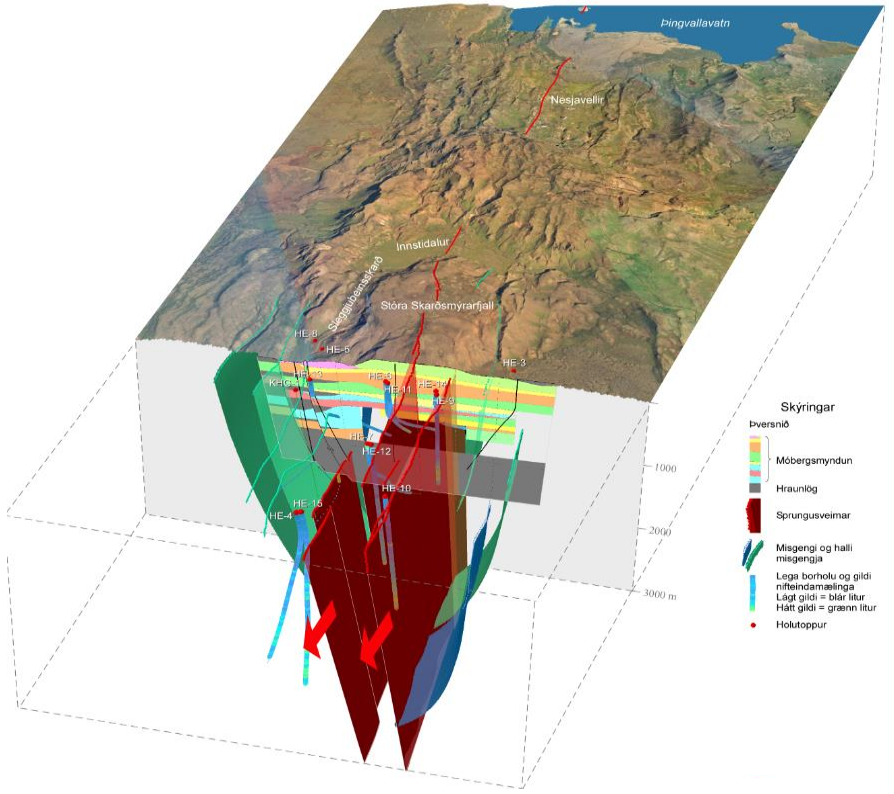
Model	*Potential variable*	*Flow variable*	*Parameters*	
			conductivity	*capacitivity*
MR-rod	velocity	force	spec. dashpot conductance	spec. mass
electric CR-line	voltage	current	spec. conductance	spec. capacitance
Magneto-telluric model	magnetic field intensity	electric field intensity	resistivity	permeability
thermal model	temperature	heat flow	heat conductivity	density by heat capacity
hydrological model, free surface	water level	flow	water conductivity	density by porosity
hydrological model, confined	pressure	flow	water conductivity	density by expansibility

(Bodvarsson, 1966)

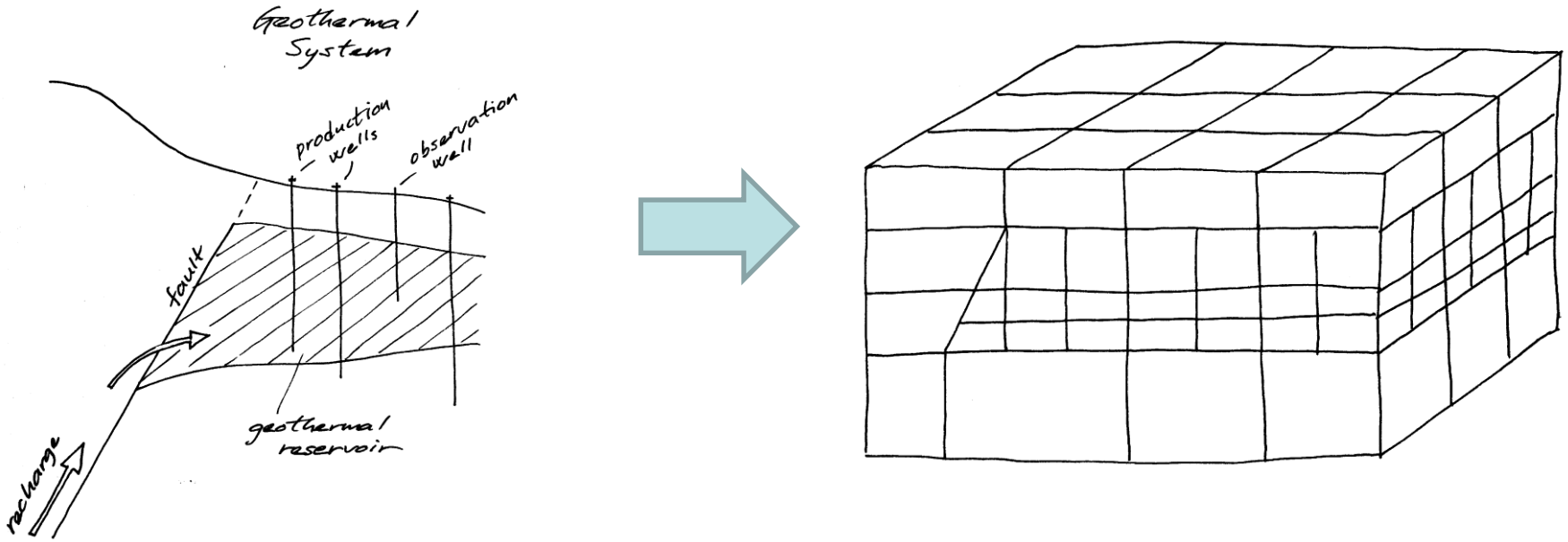
https://doi.org/10.1016/0016-7142(66)90001-9

All models are wrong, … but some are useful

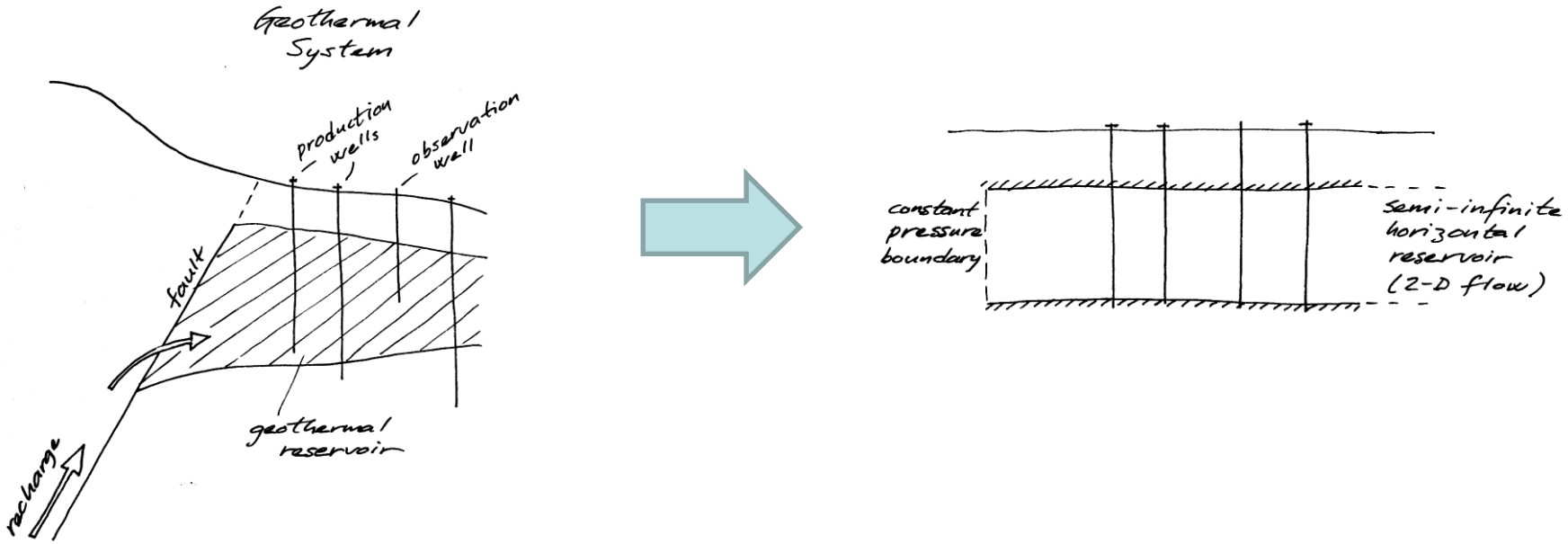




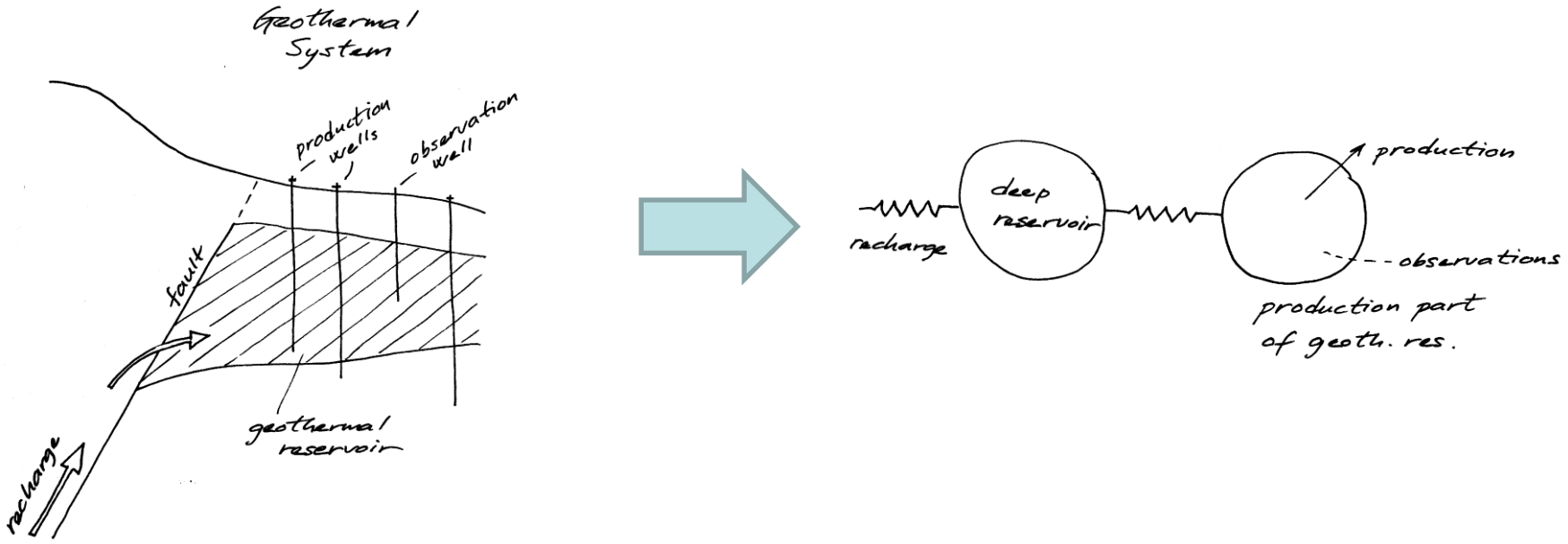
DETAILED NUMERICAL MODELS: These can accurately simulate complex geometry and structure, also variable properties. Response must be calculated by powerful computers. Such models can simulate pressure, temperature (enthalpy) and chemistry, simultaneously



SIMPLE ANALYTICAL MODELS: Geometry and structure greatly simplified; properties assumed constant. Analytical expressions describing response can be derived. Such models usually simulate either pressure, temperature or chemistry



LUMPED NUMERICAL MODELS: Geometry (mostly) ignored; properties integrated into lumped values. Response given by analytical functions



Lumped parameter modelling of geothermal systems

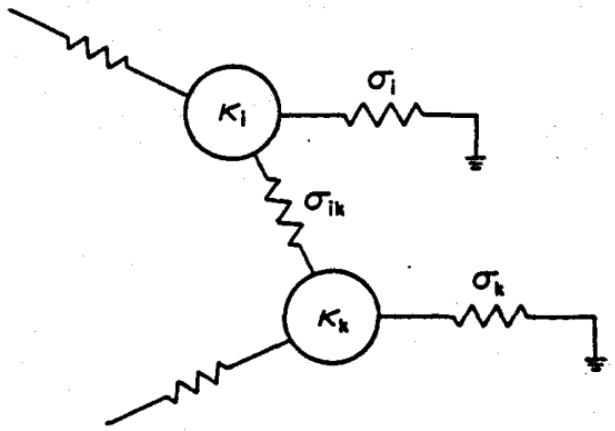


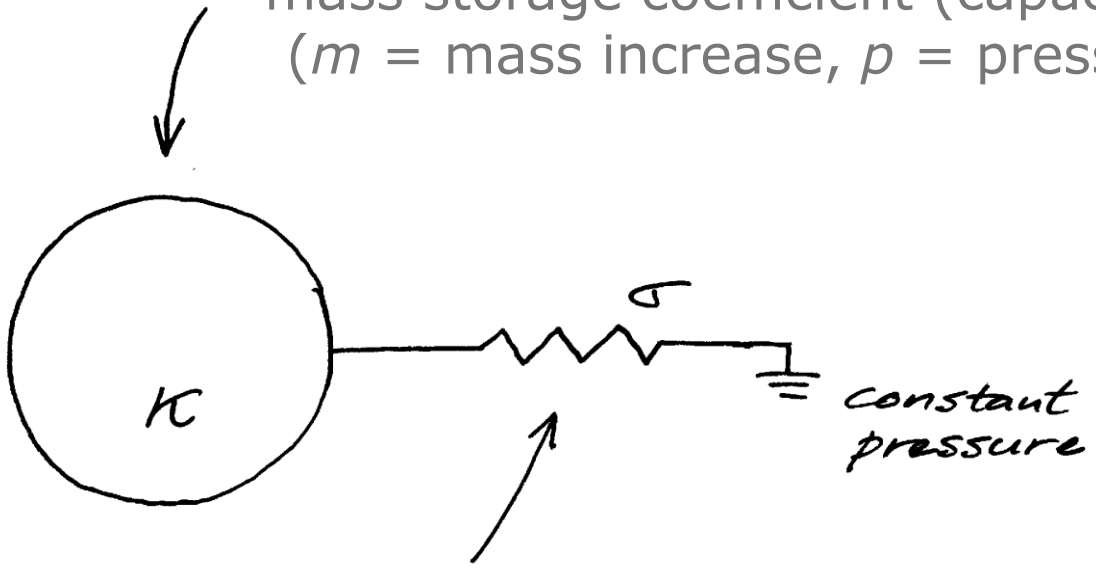
Figure 1. *General lumped capacitor/conductor network.*

- Lumped parameter modelling may be used to simulate water level/pressure change data. Can simulate long and detailed data-sets very accurately
- A method has been developed that automatically simulates data through a non-linear iterative least-squares technique

Axelsson, G., 1989: Simulation of pressure response data from geothermal reservoirs by lumped parameter models. Proceedings 14th Workshop on Geothermal Reservoir Engineering, Stanford University, USA, 257-263

Lumped parameter models – Basic components

Tank (capacitor) simulates storage in a reservoir,
mass storage coefficient (capacitance) $\kappa = m/p$
(m = mass increase, p = pressure increase)



Resistor (conductor) simulates flow resistance (permeability)
in a reservoir, conductance $\sigma = q/\Delta p$
(q = mass flow, Δp = pressure differential)

Basic components (cont.)

- **Storage coefficient**

$$\kappa = Vs$$

V = volume of the tank

s = its storativity (depends on storage mechanism)

- **Conductance** (for one dimensional flow[1)])

$$\sigma = kA/\nu L$$

L = length of resistor

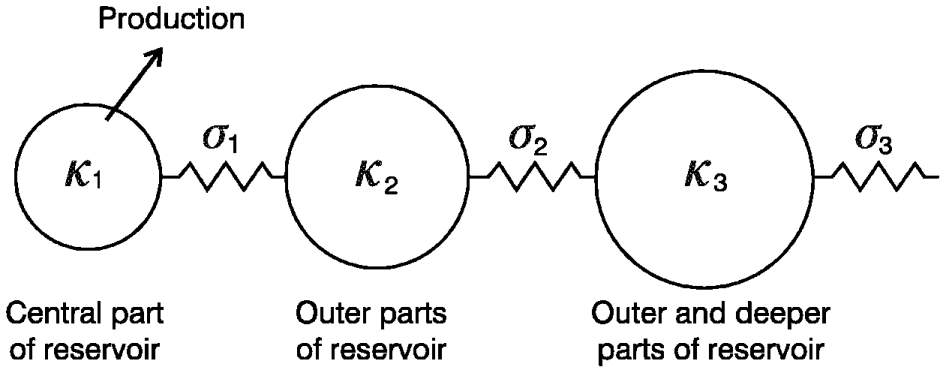
A = cross-sectional area of resistor

k = its permeability

ν = kinematic viscosity of fluid

1) depends in general on flow geometry

Linear Model

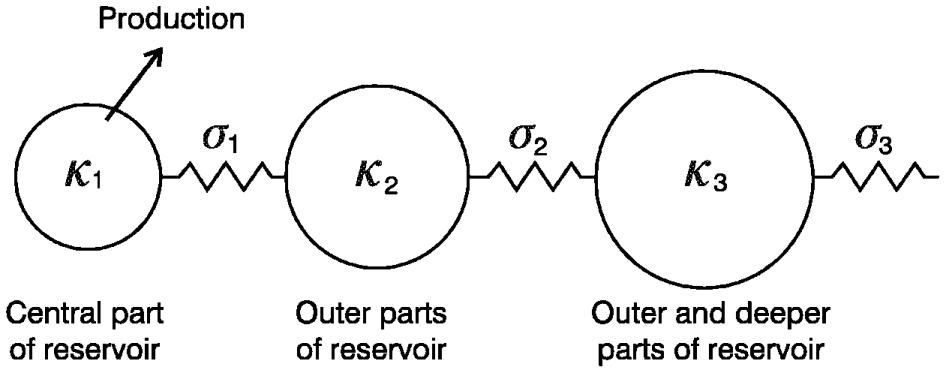


Basic equations

$$\text{Mass: } \kappa = \frac{m}{p} \quad \Rightarrow \quad m = \kappa p$$

$$\text{Flow: } \sigma = \frac{q}{\Delta \text{p}} \quad \Rightarrow \quad \text{q} = \sigma \Delta p$$

LINEAR MODEL



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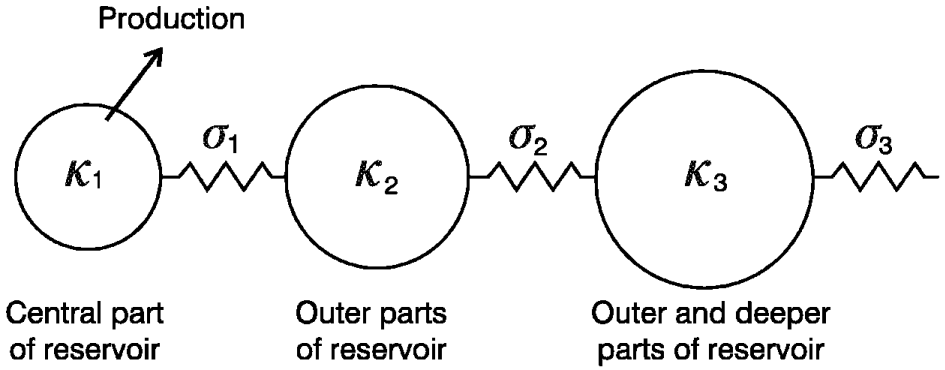
$$\text{Flow: } \sigma = \frac{q}{\Delta \text{p}} \quad \Rightarrow \quad \text{q} = \sigma \Delta p$$

Conservation of mass:

Rate of change of **mass** in tank = Flow of **mass** in/out of tank + Sink/source of **mass** in tank

Linear model



Basic equations

$$\text{Mass: } \kappa = \frac{m}{p} \quad \Rightarrow \quad m = \kappa p$$

$$\text{Flow: } \sigma = \frac{q}{\Delta \text{p}} \quad \Rightarrow \quad \text{q} = \sigma \Delta p$$

Conservation of mass:

$$\kappa_i \frac{dp_i}{dt} = \sum_{k=1}^{N} q_{ik} - \sigma_i \left(p_i - p_0 \right) - Q_i$$

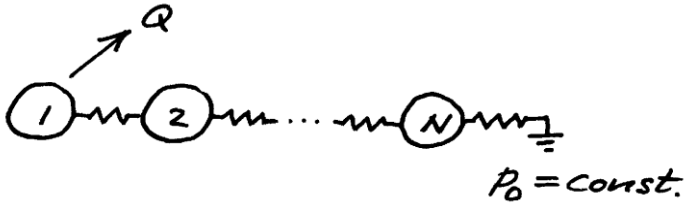
Flow between tanks:

$$q_{ik} = \sigma_{ik} \left(p_k - p_i \right)$$

Linear model – cont.

Open N-tank model (ladder)

pressure in any tank:



$$p(t) = p_0 - \sum_{j=1}^{N} Q \frac{A_j}{L_j}(1 - e^{-L_j t})$$

Closed N-tank model (ladder)

pressure in any tank:

$$p(t) = p_0 - \sum_{j=1}^{N-1} Q \frac{A_j}{L_j}(1 - e^{-L_j t}) - QBt$$

- The coefficients A_j, L_j and B are (complex) functions of the model parameters κ_j and σ_j, for j = 1,2,…N

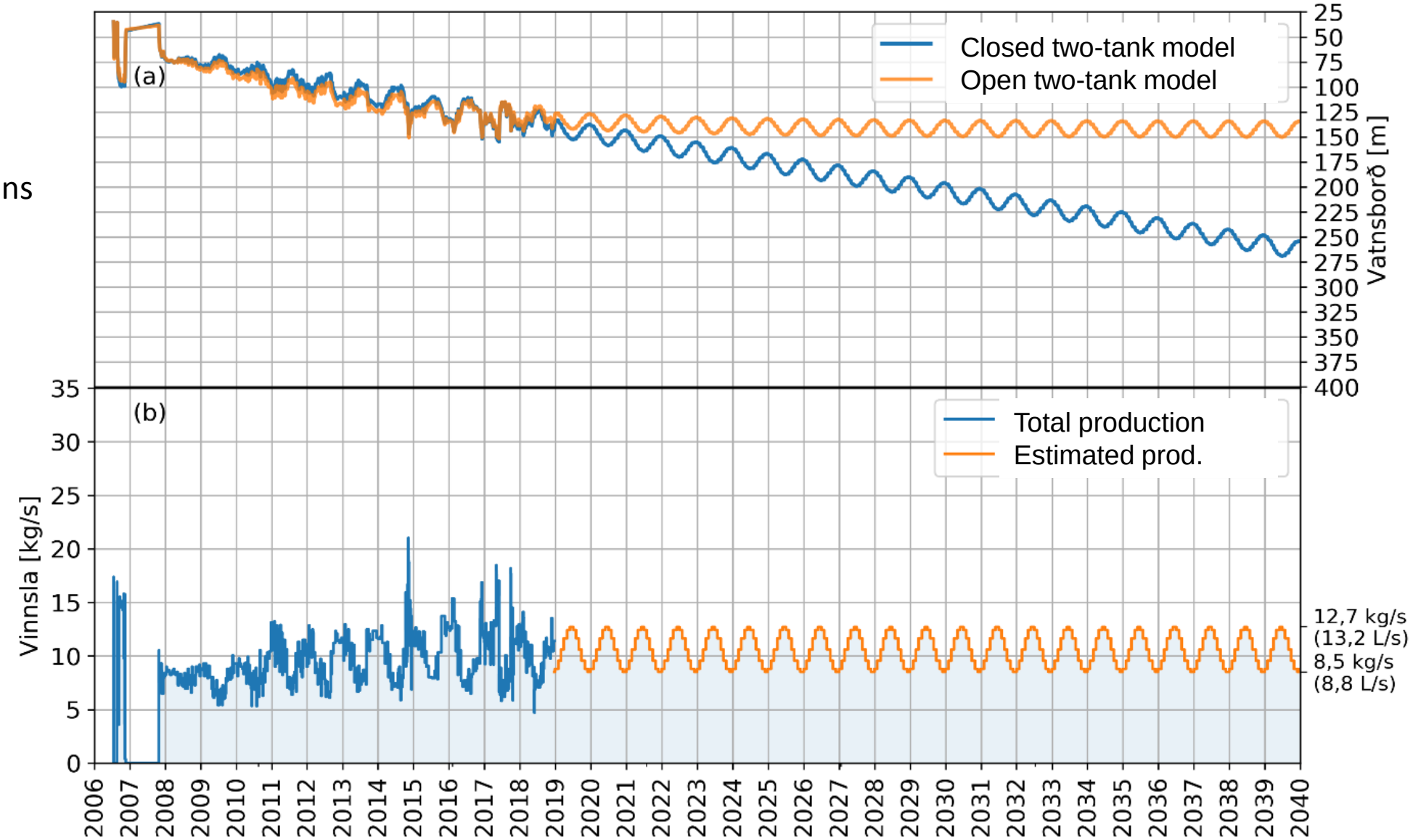
Presently implemented method

- Linear model (connections between tanks)
- Single source of production (from one tank)
- The method has proved to be very successful in predicting pressure changes
- Applied as a tool for resource assessment

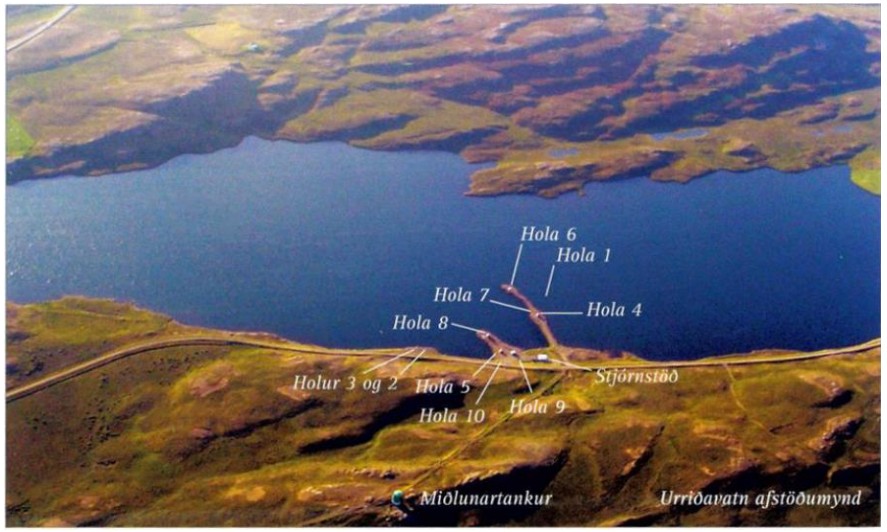
Water-level predictions by lumped parameter models

CLOSED MODEL = conservative predictions

OPEN MODEL = optimistic predictions



Other types of lumped models



Rúnar Snær Reynisson (2006). *Hitaveituævintýr Egilsstaða og Fella 1979–2006. Saga Hitaveitu Egilsstaða og Fella í rúman aldarfjórðung*. Útg. HEF. 146 s.

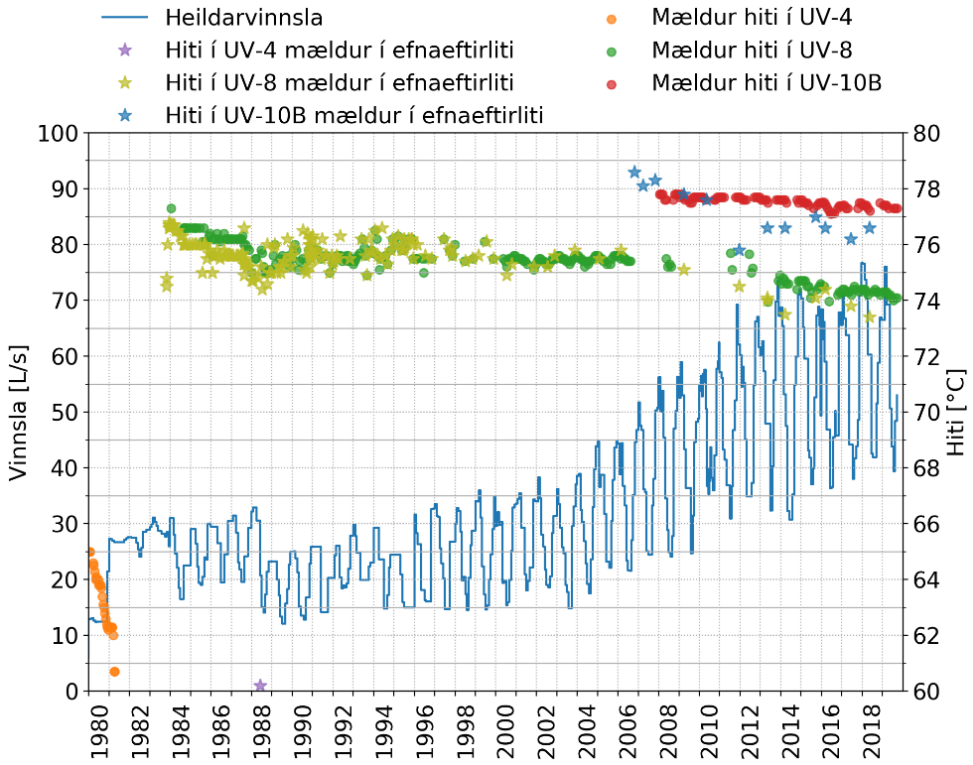
The geothermal system under Urriðavatn in the east of Iceland is taking in cold fresh water from the lake above.

The temperature correlates with changes in Cl, this can be modelled with a lumped model.

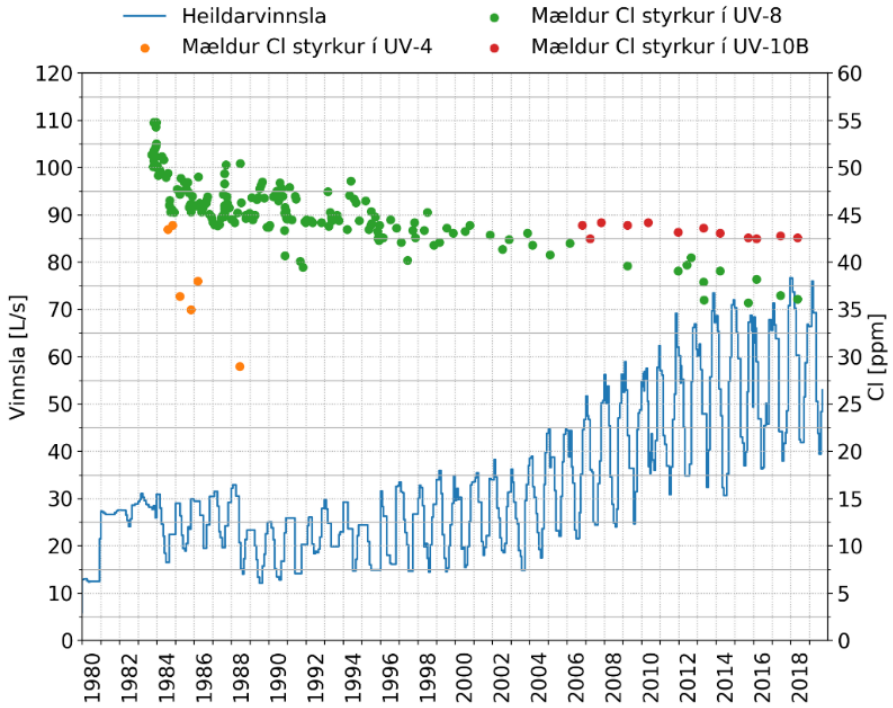
Case study by Gunnar Þorgilsson

Predicting Temperature from change in Cl

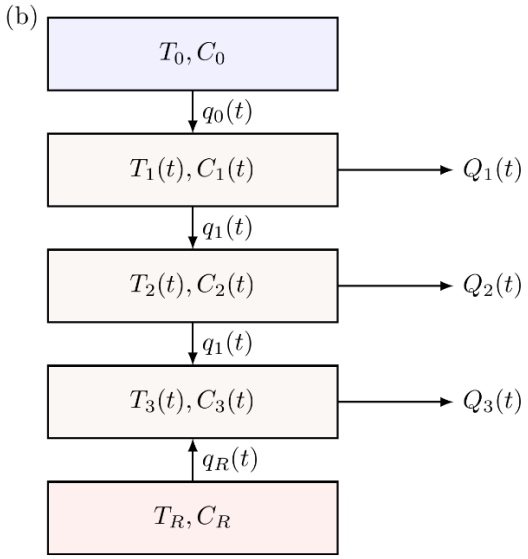
Temperature data



Cl data



Linear model for temperature/Cl concentration



Cl conservation equations

$$\rho_v \phi_1 V_1 \frac{dC_1}{dt} = q_0 C_0 - q_1 C_1(t) - C_1(t) Q_1(t)$$

$$\rho_v \phi_2 V_2 \frac{dC_2}{dt} = q_1 C_1(t) - q_2 C_2(t) - C_2(t) Q_2(t)$$

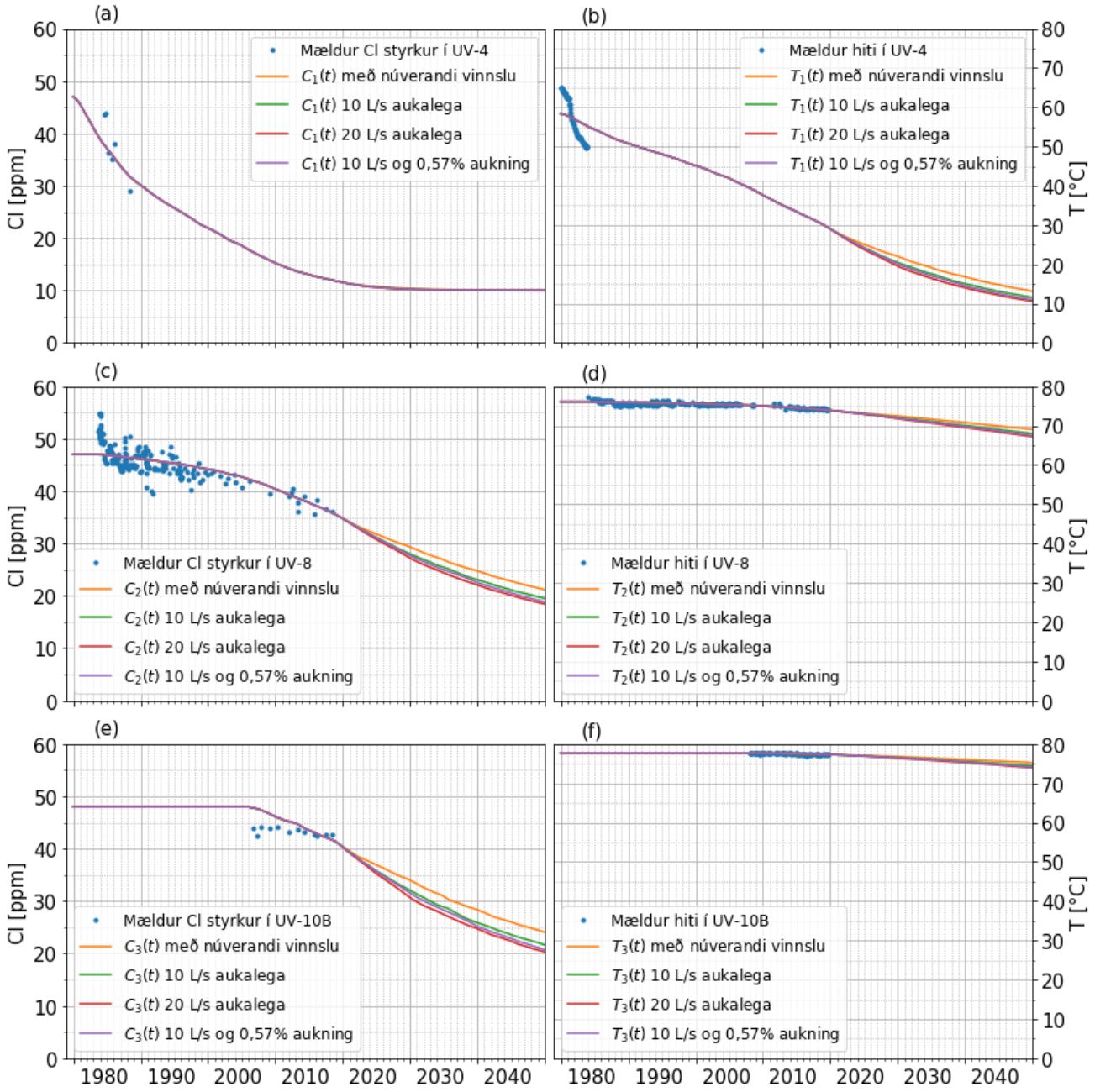
$$\rho_v \phi_3 V_3 \frac{dC_3}{dt} = q_2 C_2(t) + q_R C_R(t) - C_3(t) Q_3(t)$$

Heat conservation equations

$$\langle \rho c \rangle_1 V_1 \frac{dT_1}{dt} = c_v q_0 T_0 - c_v q_1 T_1(t) - c_v Q_1(t) T_1(t)$$

$$\langle \rho c \rangle_2 V_2 \frac{dT_2}{dt} = c_v q_1 T_1(t) - c_v q_2 T_1(t) - c_v Q_2(t) T_2(t)$$

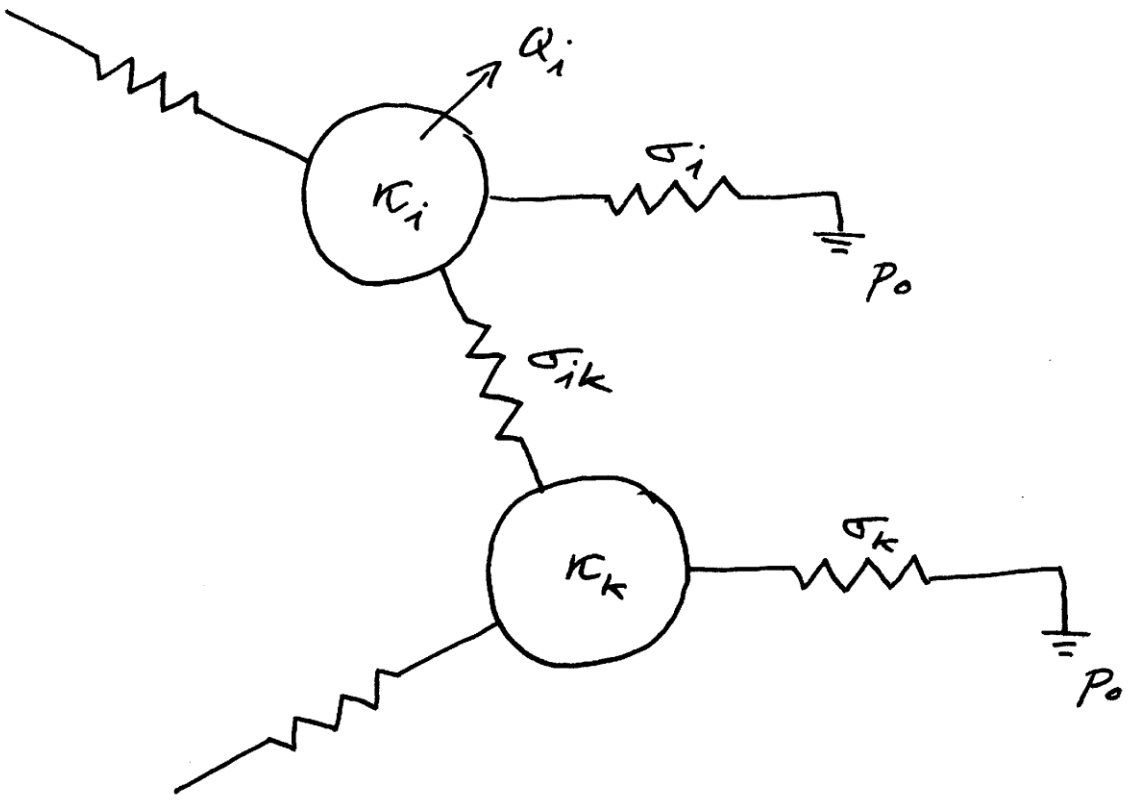
$$\langle \rho c \rangle_3 V_3 \frac{dT_3}{dt} = c_v q_2 T_2(t) + c_v q_R T_R(t) - c_v Q_3(t) T_3(t)$$



ÍSOR

From Gunnar Þorgilsson

General Model



General problem $$\mathbf{K}\frac{d\vec{p}}{dt} + \mathbf{A}\vec{p} = \vec{f}$$

Storage $$\mathbf{K} = \begin{bmatrix} \kappa_1 & & \\ & \ddots & \\ & & \kappa_n \end{bmatrix}$$

Conduction

$$\mathbf{A} = \begin{bmatrix} \sum_j^N \sigma_{1j} + \sigma_1 & -\sigma_{12} & \dots & -\sigma_{1n} \\ -\sigma_{12} & \sum_j^N \sigma_{2j} + \sigma_2 & \dots & -\sigma_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ -\sigma_{1n} & -\sigma_{2n} & \dots & \sum_j^N \sigma_{nj} + \sigma_n \end{bmatrix}$$

Equivalent eigenvalue problem

$$\mathbf{A}\vec{\tau} = \lambda\mathbf{K}\vec{\tau}$$

Pressure Response

- Impulse production

$$\vec{f}_k(t) = \begin{cases} \delta(t) & \text{if } i = k \\ 0 & \text{else} \end{cases}$$

- Impulse solution

$$\vec{h}(t) = \mathbf{T} e^{-t\mathbf{\Lambda}} \mathbf{T}^{\intercal}$$

- General production

$$\vec{f}_s(t) = \sum_{j=1}^{M} (\vec{Q}_j - \vec{Q}_{j-1}) H(t - t_j)$$

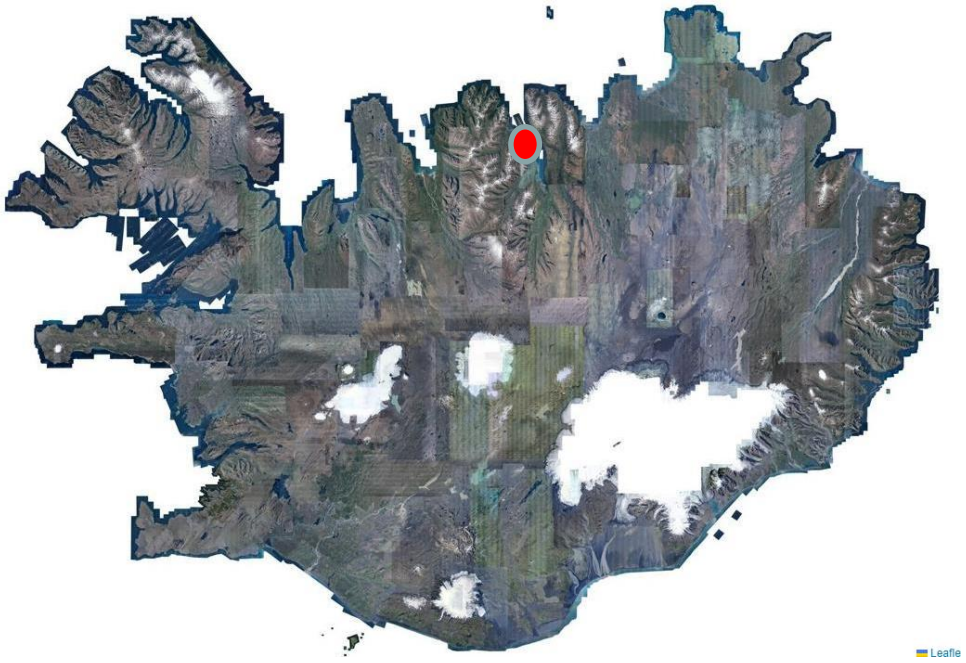
- General pressure response

$$\vec{p}(t) = \int_0^t \vec{h}(t-\tau)\vec{f}(\tau)d\tau$$

$$\vec{p}(t) = \mathbf{T} \sum_{j=1}^{M} \left[\mathbf{I} - e^{-(t-t_j)\mathbf{\Lambda}}\right] \mathbf{\Lambda}^{-1} \mathbf{T}^{\intercal} (\vec{Q}_j - \vec{Q}_{j-1})$$

(Axelsson, 1989)

Hjalteyri – North Iceland



Case study by Kjartan Marteinsson